GENERAL MATHEMATICAL AND PHYSICAL BACKGROUND OF ATTRACTOR FUNCTION

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The working paper shows that the thermal equilibrium between incoming insolation and the outgoing black body (BB) radiation results in a surprisingly high temperature of the surface (skin) temperature. The classical explanation of heating the otherwise freezing planet by greenhouse gasses as water vapour, is far too simple. Much more is going on. Due to the opacity of the atmosphere for IR radiation, explaining the window for IR radiation observed by satellites, the temperature can rise even above the boiling point of water without additional cooling. That is already a remarkable result.

A cooling mechanism is necessary to get the temperature in accordance with the observations. The incoming insulation[1] and outgoing BB must be cooled to a daily averaged temperature T_{aver} corresponding to the observations. In the paper the cooling is represented mathematically by an attractor function known from complexity theory (or chaos theory). The function is (see discussion around equation 4 in paper):

$W_{ec} = W_c (T^n/T_d - 1)$, with $1 \leq n<<2$,with variable T and parameter T_d in degree C.

The function is representing the very complex physics of the cooling processes. This cooling is referred to as the wind-water effect.

The SDC algorithm coded as a MS Excel spreadsheet has been recoded into Python 3 scripts, which has two main advantages. The diurnal cycle can be extended beyond five days to an arbitrary number of days. Secondly the trial and error approach of finding the parameters of the attractor function can be incorporated into a Python (recursive) algorithm. The algorithm converges rapidly and results into an estimation of W_c given T_d. The requirement is that the diurnal cycle is in "balance", meaning the temperature difference at the beginning of the two last days is below a given very small threshold.

Another intriguing result of the paper is the outcome of the change in temperature due to increase of the opacity with a factor 0.01 (e.g. representing a doubling of CO_2, see also chapter 21). For the clear sky situation (g = 1, see the paper for 30 N day 81 in chapter 21)

	f	g	rWW	Taver	ΔrWW	ΔTaver
Balanced	0.68	1.0	106.4716	23.30002		
Unbalanced	0.69	1.0	108.6599	23.63679	2.1883	0.336774
Balanced	0.69	1.0	110.3982	23.30009	3.9266	7.0E-5

By introducing "clouds" for 5 days (reducing both the insolation and the atmospheric window) (arbitrarily chosen values are: g = 0.4, f = 0.9), we see a similar result:

	f	g	rWW	Taver	ΔrWW	ΔTaver
Balanced	0.90	0.4	53.5835	23.30010		
Unbalanced	0.91	0.4	54.9272	23.38286	1.3437	0.08275
Balanced	0.91	0.4	57.5102	23.30014	3.9267	0.00013

[1] *R. Clark, Private Communication, 2017. Solar flux calculations based on IEEE Standard 793 'IEEE Standard for Calculating the Current-Temperature Relationship of Bare Overhead Conductors' IEEE, June 1993. Clear Atmosphere Coefficients, page 13.*

Two things can be observed for this example. As expected the required cooling is smaller with clouds than in clear sky. The ΔrWW due to increase of opacity is however not different between clear sky and clouds, while the increase in temperature is neglectable in both cases. The impact of a change in opacity is apparently only affecting the required rWW with a value of around 4 W/m^2, and not the temperature.

We may wonder if we can define more general mathematical conditions for that attractor function. The function depends of course on the temperature.

There are three obvious options for the function:

1) **Function has a constant value:**
   If W_{ec} = constant during the day, the cooling is enough to reduce the heat to the observable. The diurnal cycle is soon in balance, especially on land conditions. The constant is found by trial and error.
2) **Arbitrary function without zeros.** That will work if the (diurnal) average of W_{ec} is equal to the constant cooling of option 1)
3) **Function with Zeros**: That will work if the zeros are far away of the observed temperature, and with the condition that the function average over last day, is again the constant defined in 1)

The attractor function used in the paper is an example of option 3.

If n is zero, we fall back on option 1). If n > 1, the zero shifts more towards T = 0 C, but no major difference from the case if n = 1.

Investigating the W_{ec} function as function of T_d, a complication is that $T_d = 0$ must be avoided. That coordinate singularity results in very large values for Wec.

In Figure 1 the dependency of T_d is plotted using the above recursive algorithm for W_c. The W_{ec} function is zero when T is T_d, while the averaged W_{ec} is constant. That indicates a singularity in W_c as function of T_d, nicely shown in Figure 2.

The observed temperature of 60 N day 81 according to Table III of the paper is 4.1 C. Hence W_{ec} as function of T_d has a zero point for $T = T_d \approx 4.1$. If the daily temperature is varying mainly around T = 4.16, the W_c must be very large to generate a W_{ec} of 1.5858 W/m^2. If Td is far away of observed temperature, the singular behaviour is not occurring. The coordinate singularity near T_d = 0 C is visible too.

If Wc becomes very large near the singularity, the varying daily temperature becomes instable (because no Wec can be generated with a value of 1.58 W/m^2). The attractor W_c function fails to “deliver” the cooling required near a zero point. Worse the temperate change due to W_{ec} on time t + Δt is much larger than at time t, and T diverges rapidly as function of time. At the other hand, when T_d is not near 4.16, the averaged W_{ec} is independent of T_d.

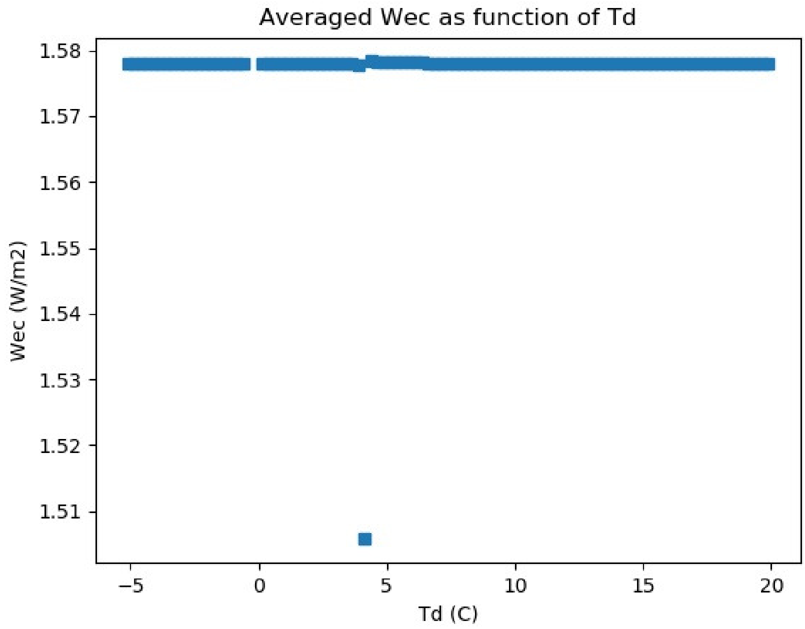

Figure 1 Averaged W_{ec} as function of T_d for 60 N day 81 (ocean with n = 1). T_d in steps of 0.25 between -5.1 and 20.1 C. Anomalous values (> 2.0) around T_{aver} of 4.1 are removed from plot (see also Figure 2). Note the "constant" value of W_{ec} (averaged) ≈ 1.5880.

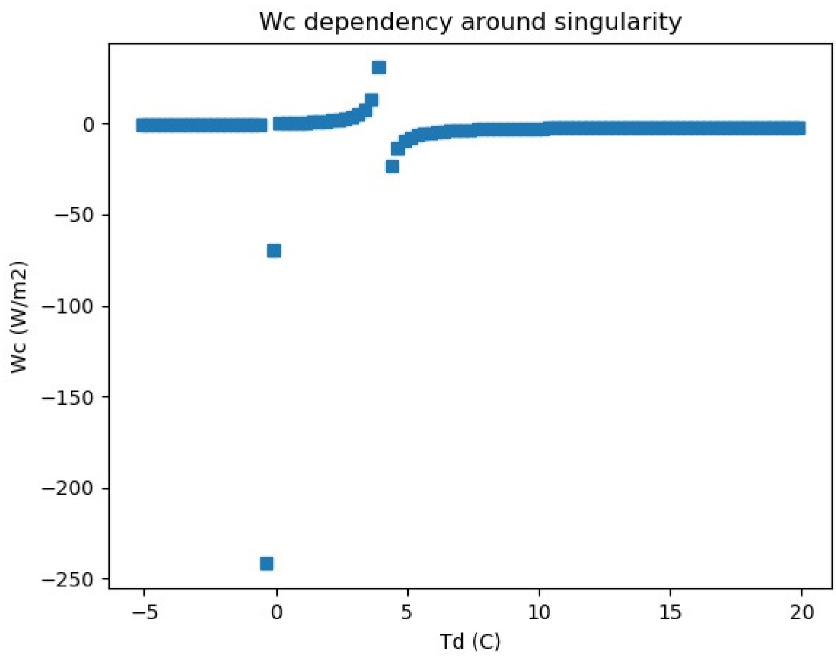

Figure 2. Wc as function of T_d for 60 N day 81around the singularity near $T \approx 4.16$ W_c is positive for $T_d < 4.16$ C and negative for $T_d > 4.16$ C. Note extreme large values of $W_c > 2000$ near the singularity are removed from plot. Large negative values are seen for $T_d \approx 0$.

Physicists have a hate-love relation to complexity theory. They prefer in general a reductionistic approach in physics. They want to unravel the "real" physics behind a complex process. To unravel the physics behind climate, one enters often into the world of climate modelling with great disadvantages due to lateral and vertical resolution of grid cells and temporal resolution of the finite element techniques used. The real physics "happens" at a scale of a few mm's and instantaneous in comparison with time steps in GCM's (impact of gravity, heat and mass transport). That raises "discussions" about assumption of LTE or non-LTE regarding incorporation of radiation transfer processes in those models.

Contrary to the partly 4D vectorially nature of the mass and heat transfer equations of GCM, a simple scalar cooling function in the autonomous regulatory process, enables to describe the missing link between the observed temperature and transport of required heat. The paper shows that this works for given locations and date.

The phase changes of water are instrumental to heat transport to and from the skin and troposphere. The W_{ec} function parameter T_d is chosen to be the "dew point" of water. However, this does not prove that this likely attractor function must be also the only function possible. Strictly mathematically any function with the required averaged cooling does the job. Tests shows, using the Python code, that simple functions like sine of cosine functions also work. Only the cooling in situ has been studied. Complexity of heat transport to and from different latitudes is not studied and may need adapted or additional attractors to study further the autonomous regulatory processes of the water planet.